MATHEMATICS DEPARTMENT

"A computer is the mathematicians best friend"

μ - Games
Mathematics Utrecht

October 2022

Rules:

The idea of this event is to gap the bridge between mathematics and programming. When working on these exercises, we hope the participant will get a better understanding of the underlying mathematical concepts. You will not be required to do a lot of difficult programming. With array manipulation and basic functionality, you should be able to solve all the exercises.

When working on these exercises, you must conform to the following rules.

- You are allowed to work in groups of maximum 4 persons.
- You will have 2.25 hours time.
- You can use no software packages.
- You can not look up any computer code that may help you solve the problem.

After the three hours, the solutions to the exercises will be discussed. To check your own solution, one can go to the website *http://clover.science.uu.nl/dj*.

Problem 1: Fermat Theory

Difficulty: ★ ☆ ☆ ☆ ☆
Key words: Fermat theory, Polynomials

Let us consider an arbitrary polynomial $f \in \mathbb{Z}[x]$, i.e. a polynomial in x with coefficients in $\mathbb{Z}$. Then, $f(x+y)$ is a polynomial in $\mathbb{Z}[x,y]$. In particular, we can uniquely decompose this polynomial as $f(x+y) = g(x) + yh(x,y)$. We now want to know what the polynomial $h(x,0)$ is with coefficients modulo 10^9+7.

Input

- One line with the integer $0 \leq n < 10^5$, the degree of the polynomial.
- One line with $n+1$ space-separated integers a_k such that $-10^9 < a_k < 10^9$ indicating the coefficients of f, ordered from the 0th degree to the nth degree.

Output

- One line with a positive integer k the degree of the polynomial $h(x,0)$.
- One line with the space-separated coefficients of the polynomial $h(x,0)$ ordered from 0th degree to the kth degree.

Example

Input	Output
0 1	0 0

Input	Output
2 12 2 3	1 2 6

Problem 2: Hilbert Hotel Lite

Difficulty: ★ ☆ ☆ ☆ ☆
Key words: Number Theory, Primes

You want to sleep a night in Hilbert's hotel, but you know you can only sleep well in a prime numbered room. All the rooms are filled up sequentially, so the first person arriving gets room 1, while the second person gets room 2, etc.

When you ask at the door how many people are in yet, they will not only give you the number n of visitors already in the hotel, but also in how many ways they could have been assigned to the n rooms. But as this second number gets really big, they will only tell you the number modulo $n+1$.

If you go in now, will you be able to sleep well tonight?

Input

- One line with the integer $0 < n < 10^{35}$, the number of visitors already in the hotel.
- One line with the number of ways to assign the n visitors to n rooms, modulo $n+1$.

Output

- The integer 1 when you will get a prime-numbered room, and the integer 0 when you will not get a prime-numbered room

Example

Input	Output
2 2	1

Problem 3: Limit Map

Difficulty: ★ ★ ★ ☆ ☆
Keywords: Ergodic Theory, Dynamical Systems

We define the discrete recursive equation for $0 < x < 1$ a rational number.

$$f_{n+1}(x) = 1 + \frac{x^2 - 1}{1 + f_n(x)}, \text{ with } f_0 = 0. \tag{1}$$

In this exercise, we will look at the composition of this map together with the following map.

$$\Gamma : \mathbb{T}^2 \to \mathbb{T}^2, \tag{2}$$
$$(x, y) \mapsto (2x + y, x + y) \bmod 1. \tag{3}$$

We are interested in the behavior of the map $\Gamma^k(\lim_{n\to\infty} f_n(x), \lim_{n\to\infty} f_n(y))$ under choices of $x, y \in \mathbb{Q}$ and $k \in \mathbb{N}$.

Input

- A line with space-separated integers $-10^6 < p < 10^6$ and $-10^6 < q < 10^6$, such that $x = p/q$.
- A line with space-separated integers $-10^6 < w < 10^6$ and $-10^6 < r < 10^6$, such that $y = w/r$.
- An integer $c \in \mathbb{N}$ such that $k = 10^c$.

Output

- Two integers l and k such that $\Gamma^k(\lim_{n\to\infty} f_n(x), \lim_{n\to\infty} f_n(y))_0 = l/k$, where the index 0 indicates the first coordinate.
- Two integers b and v such that $\Gamma^k(\lim_{n\to\infty} f_n(x), \lim_{n\to\infty} f_n(y))_1 = b/v$, where the index 0 indicates the first coordinate.

Examples

Input	Output
1 3	2 3
2 5	2 5
1	

Problem 4: Zeckendorf Cover

Difficulty: ★ ★ ★ ☆ ☆
Key words: Partitions, Combinatorics, Fibonacci Numbers

Zeckendorf's Theorem states that any positive integer can be decomposed uniquely as a sum of distinct, non-consecutive Fibonacci numbers.

We call a set of positive integers a Zeckendorf n-cover if the collection of their Zeckendorf decompositions is pairwise disjoint and the union of this collection is equal to the first n Fibonacci numbers. Here, we take $F_1 = 1$ and $F_2 = 2$. For example, the set $\{7, 12\}$ is a Zeckendorf 5-cover.

We now want to know for a given $n \in \mathbb{N}$: how many Zeckendorf n-covers are there modulo $10^9 + 7$?

Input

- One line with the integer $0 < n < 1000$.

Output

- One integer with the number of Zeckendorf n-covers modulo $10^9 + 7$.

Example

Input	Output
5	15

Input	Output
10	21147

Problem 5: Graph Magic

Difficulty: ★ ★ ★ ★ ☆
Key words: Graphs, Matrices, Eigenvalues

The adjacency matrix A of a graph $G = (\{v_i \mid i \in \{1, 2, \ldots, n\}\}, E)$, is defined as

$$A_{ij} = \begin{cases} 1 & \text{if there is an edge between vertex } v_i \text{ and vertex } v_j, \\ 0 & \text{else.} \end{cases} \tag{4}$$

The eigenvalues of a graph G are defined as the eigenvalues of its adjacency matrix. A set of eigenvalues $\lambda_1, ..., \lambda_n$ with multiplicity $\mu_1, ..., \mu_n$ is called symmetric, whenever for all $1 \leq i \leq n$, we have that $-\lambda_i$ is also an eigenvalue with multiplicity μ_i.

In this exercise, you are given an adjacency matrix A and you have to determine whether or not the eigenvalues are symmetric.

Warning: you are not allowed to use numpy or alike libraries to calculate these eigenvalues.

Input

- One line with the integer $1 < n < 10^5$, the amount of vertices in G.
- One line with the integer $1 < m < 10^7$, the number of edges in G.
- m lines with two space-separated integers i, j such that $A_{ij} = 1$.

Output

- The integer 1 when the eigenvalues are symmetric. The integer 0 when the eigenvalues are not symmetric.

Example

Input	Output
2 1 1 2	1

Input	Output
4 4 1 2 1 4 2 4 3 4	0